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Boundary-Layer Control for Increasing Lift by Blowing

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Nomenclature

x = distance along the wing section surface
y = distance from the wing section surface
c = wing chord
s = slot width
Re = Reynolds number $(= U_\infty c/\nu)$
ρ = density of air
v_j = velocity of the jet
U_∞ = freestream velocity
$U(x)$ = velocity distribution over the wing section
$\bar{U}_K$ = mean value of the velocity on the deflected flap
$u(y)$ = velocity distribution in the boundary layer
ϑ = momentum loss thickness $= \int_0^\infty \rho u(y)/\rho_\infty U_\infty \,[1 - u(y)/U_\infty]dy$
c_μ = momentum coefficient $= 2[(\rho_j v_j^2 s)/(\rho_\infty U_\infty^2 c)]$
$c_{\mu S}$ = minimum momentum coefficient required for preventing boundary-layer separation
TE = trailing edge
S = point of separation

It is well known that the actual lift of a wing with a deflected trailing edge flap is far below the lift predicted by potential flow theory because of boundary-layer separation. This separation can be prevented by blowing a high velocity air jet out of a narrow slot near the flap knee into the boundary layer (see Fig. 1a). The minimum momentum coefficient required for avoiding boundary-layer separation and for thus attaining the theoretical lift has so far only been found by experiments.[1] In the following, a simple method will be described for calculating the momentum coefficient necessary to prevent separation. This is achieved by boundary-layer calculations, using some empirical results from detailed measurements on boundary layers with blowing.

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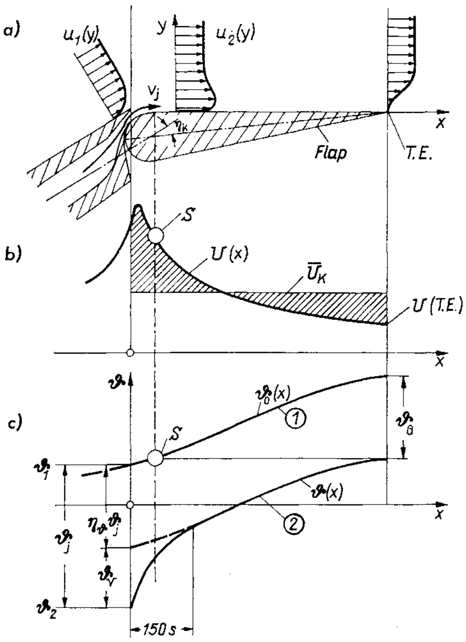

Fig. 1 Boundary-layer control by blowing over a deflected trailing edge flap. a) Boundary-layer profiles on the flap. b) Velocity distribution on the flap from potential flow theory. c) Distribution of momentum loss thickness along the deflected flap (① without blowing; ② with blowing).

At the point where the jet is blown into the boundary layer, the momentum loss thickness of the boundary-layer ϑ changes abruptly, as is indicated in Fig. 1c. This jump in momentum loss thickness is

$$\vartheta_j = \vartheta_2 - \vartheta_1 \qquad (1)$$

where ϑ_1 and ϑ_2 are the momentum loss thicknesses just in front of the slot and behind the slot, respectively.

From simple momentum considerations, the following relation between c_μ and ϑ_j is found[2]:

$$\vartheta_j = -\tfrac{1}{2}c_\mu c\,[1 - (U_\infty/v_j)] \qquad (2)$$

One part ϑ_v of the "equivalent momentum thickness ϑ_j" of the jet is lost by friction and mixing and only the remaining part $\eta_\vartheta\vartheta_j$ is actually available for boundary-layer control. In order to get information on these losses, detailed boundary-layer measurements were carried out for different jet velocities v_j and different constant mainstream velocities U_∞.[2,3] In all cases within the range $v_j/U_\infty \geq 2$, which was covered by these tests, the momentum thickness $\vartheta(x)$ followed the pattern shown in Fig. 1c. That is, behind the jump of the amount ϑ_j at the slot, the momentum thickness rises very steeply within a distance of about 150 s from the slot. It then follows a curve that is parallel to the curve $\vartheta_0(x)$ of the corresponding case without blowing. From these measurements the following general law was found for the "efficiency factor" η_ϑ of the jet:

$$\eta_\vartheta = 0{,}85\,[1 - (U_\infty/v_j)] \qquad (3)$$

In the procedure utilized to find the theoretical estimation of the minimum momentum coefficient required for preventing boundary-layer separation, the next step is the calculation of the potential flow velocity distribution of the wing section with the deflected flap and, thus, with a finite suction peak at the flap knee.[4] By applying boundary-layer calculations to this velocity distribution,[5] the separation point S in the case of no blowing can easily be found (see Fig. 1b). In order to obtain the lift predicted by potential flow theory, the separation point has to be shifted to the trailing edge by blow-

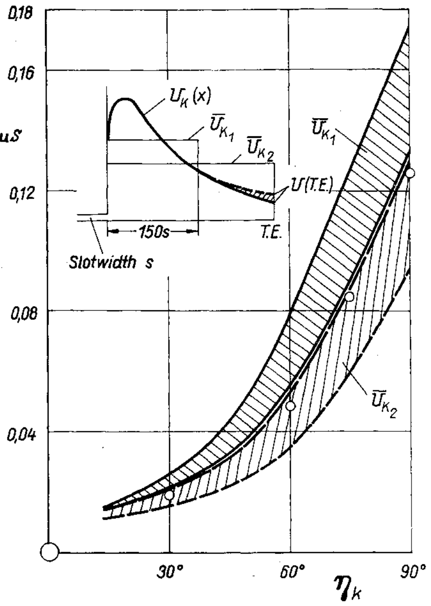

Fig. 2 Momentum coefficient $c_{\mu S}$ for preventing separation from experiment and theory. Slot width $s/c = 0.8 \times 10^{-3}$, ○ = experiment, —— = theory with $\bar{U}_K = \bar{U}_{K1}$, and --- = theory with $\bar{U}_K = \bar{U}_{K2}$. Hatched area represents scatter due to uncertainty of the value of U at the trailing edge.

ing. This is achieved when the net amount of the blown-in jet momentum $\eta_\vartheta \vartheta_i$ is equal to the increase in the momentum thickness ϑ_G of the boundary layer with no blowing which occurs between the separation point S and the trailing edge (see Fig. 1c). Thus,

$$-\eta_\vartheta \vartheta_{iS} = \vartheta_G \tag{4}$$

The increase in momentum thickness ϑ_G can easily be obtained by calculating the turbulent boundary layer as described in Ref. 5:

$$\frac{\vartheta_G}{c} = \frac{0{,}037}{Re^{1/5}} \left[\frac{U(TE)}{U_\infty}\right]^{-3} \left\{\int_S^{TE} \left[\frac{U(x)}{U_\infty}\right]^{3.5} d\left(\frac{x}{c}\right)\right\}^{0.8} \tag{5}$$

The expression for the minimum momentum coefficient required for preventing boundary-layer separation is finally found by inserting Eqs. (2) and (3) into Eq. (4):

$$c_{\mu S} = 2\,\frac{\vartheta_G}{c}\,\frac{1}{0.85\,[1 - (U_\infty/v_i)]^2} \tag{6}$$

with ϑ_G according to Eq. (5). This equation indicates that the required momentum coefficient $c_{\mu S}$ decreases with increasing jet velocity and with decreasing slot width. This is in agreement with the experimental results of different authors which are compared in Refs. 2 and 6.

For the calculation of the flapped wing in Eq. (6), it is reasonable to use the mean velocity $\bar{U}_K$ on the flap (see Fig. 1b) instead of the mainstream velocity U_∞. Thus

$$c_{\mu S} = 2\,\frac{\vartheta_G}{c}\,\frac{1}{0.85\,[1 - (\bar{U}_K/v_i)]^2} \tag{7}$$

By this method, which is described in more detail in Ref. 2, a good agreement with measurements has been achieved. A comparison between experiment and theory is given in Fig. 2. The hatched areas indicate how the uncertainty in the calculation of the velocity at the trailing edge $U(TE)$ and the definition of the mean value $\bar{U}_K$ influence the results. This method has also been extended to airfoils using boundary-layer control by blowing at the leading edge.[7]

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Flow of Viscoelastic Maxwell Fluid in a Circular Pipe

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RICHARDSON and Tyler[1] investigated experimentally the flow of a Newtonian, viscous fluid in a circular tube under a periodic pressure gradient, and Sexl[2] investigated the same problem theoretically. Sanyal[3] discussed the same problem under a pressure gradient, rising as well as falling exponentially with time. The object of this note is to study the flow of viscoelastic Maxwell fluid in a tube of circular section, following Sanyal, in two cases: 1) when the pressure gradient rises exponentially with time, and 2) when the pressure gradient falls exponentially with time.

Following Bagchi,[4] the equations of motion are

$$\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{1a}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{1b}$$

where

$$\nu = \mu/\rho \tag{1c}$$

Equation (1b) shows that pressure p is a function of z and t only.

Case I: Pressure Gradient Rising Exponentially with Time

Let us assume for the pressure gradient

$$-(1/\rho)(\partial p/\partial z) = ke^{\alpha^2 t} \tag{2a}$$

and for the velocity

$$w = f(r)e^{\alpha^2 t} \tag{2b}$$

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